1 General equilibrium

Pareto Optimum

A feasible allocation x^* is called Pareto Optimal if no another feasible allocation x such that :

$$\forall i \quad u_i(x_i) \geq u_i(x_i^*)$$

$$\text{some} \quad u_i(x_i) > u_i(x_i^*)$$

Core

The core of an exchange economy with endowment e , denoted by $C(e)$, is the set of all unblocked feasible allocations. In other words, an allocation $x \in F(e)$ is in core if no subset of individuals can block it.

Walrasian equilibrium

A competitive (Walrasian) equilibrium is a pair (p, x) where x is a competitive allocation for p

2 Game Theory

Nash equlibrium

A pure action profile $a^* = (a_1^*, \ldots, a_N^*)$ is a Nash equilibrium if for each player i,

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \text{ for every } a_i \in A_i$$

That is $a_i^* \in BR(a_{-i}^*)$ for every $i \in N$
A mixed action profile $\sigma^* = (\sigma_1^*, \ldots, \sigma_N^*)$ is a Nash equilibrium if for each player i,

$$BR(\sigma^*) \in BR(\sigma_{-i}^*)$$

That is $a_i \in BR(\sigma_{-i}^*)$ for every $a_i \in \text{Supp}(\sigma_i^*)$.

Belief

An assesment is strategy/condtional **belief** pair (σ, μ) where the function $\mu : X \to [0, 1]$, gives conditional beliefs of each information set including the trivial ones.

Sequential rationality

An assessment (σ, ν) is ""sequentially rational" if playing σ_i maximizes expected utility given μ for each player i at each of player i's information sets

PBE

An assessment (σ, μ) is PBE, if

- it is sequential rational , and
- belief μ one given by Bayes's rule applied to Nature's move and to σ "Whenever possible"

BNE

Let T_1 (a finite set) be the set of possible types for player 1 and T_2 be the set of possible types for player 2 . We define (s_1^*, s_2^*) to be a BNE if for every type $t_i, s_i^*(t_i)$ solves

$$\max_{a_i \in A_i, t_j \in T_j} u_i\left(a_i, s_{-i}^*(t_j), t_j\right) \Pr_i\left(t_j \mid t_i\right)$$

$\Pr_i(t_j \mid t_i)$ is player i 's belief that the probability j is of type t_j given that i is type t_i.

3 Mechanism design and matching

direct mechanism

A direct mechanism is one in which $S_i = \Theta_i$ for each player i that is under a direct mechanism players are asked to report their types.

Individual rationality (IR)

A matching is IR if for **no** $i \in M \cup W$, such that

$$\varnothing P_i \mu(i)$$

No blocking pairs (NBP)

A pair (m, w) block μ if

- $w P_m \mu(m)$
- $m P_w \mu(w)$

both two sides in this pair has no better options than their match. -> NBP
A matching μ is stable if it is IR and there is no pair man-woman that block μ.

4 Social Choice

Social choice function

$$f(R_1, \ldots, R_N) = R$$

, where R_i is the (weak) preference of the society. (This implies indifference may occur) and we use P_i denote the strict preference R is the soical preference

Arrow's axioms

- Unrestricted domain (**UD**): The social choice function f must consider any possible combination of individual preferences over X and gives an outcome.
- Weak Pareto principle (unonimity, everyone agrees): if xR_iy for $\forall i$ then xRy
- Independence of irrelevant alternatives (**IIA**) Let $R = f(R_1, \ldots, R_N)$ and $\tilde{R} = f(\tilde{R}_1, \ldots, \tilde{R}_N)$ and $x, y \in X$. If every individual rank x and y the same way under R_i and $\tilde{R}_i$, then society rank x and y the same way under R and $\tilde{R}$.
- No dictatorship (**ND**) :There is no individual i such that for all $x, y \in X$, $xP_iy \Rightarrow xPy$ regardless of the preferences of the other individuals.