Beyond Truth-Telling: Preference Estimation with Centralized School Choice and College Admissions

AER, 2019

Gabrielle Fack, Julien Grenet, and Yinghua He

INTRODUCTION

- The DA mechanism is strategy-proof, meaning that we typically assume students should rank schools truthfully in their rank-order lists (ROLs).
- This assumption might be restrictive, and hard to hold in real-world settings, particularly when students have limited uncertainty about admission outcomes.
- For example, a student may “skip the impossible” and choose not to apply a highly selective that expects a zero admission possibility.

INTRODUCTION

This paper...

- challenges the truth-telling assumption in deferred acceptance mechanisms and proposes stability as a more realistic assumption for estimating student preferences, particularly in strict-priority settings where students may strategically omit certain schools.
- develops new empirical methods to estimate preferences.

Contribution:

- Clarify the implications of the truth-telling assumption,
- Propose a set of novel empirical approaches that are theoretically founded,
- Evaluate the performance of each approach based on simulated and real-life data.

Model

- A finite set of schools, $\mathcal{S} \equiv \{1, \ldots, S\}$, and a set of students.
- Each student i has her type: $\theta_i = (u_i, e_i) \in \Theta \equiv [0,1]^S \times [0,1]^S$.
- Utility of being assigned to schools: $u_i = (u_{i,1}, \ldots, u_{i,S}) \in [0,1]^S$.
- Priority indexes at schools: $e_i = (e_{i,1}, \ldots, e_{i,S}) \in [0,1]^S$.
- The continuum economy with a unit mass of students: $E = \{G, q, C\}$.
- G is an atomless probability measure over Θ representing the distribution of student types.
- $q = (q_1, \ldots, q_S)$ are masses of seats available at each school, $q_s \in (0,1)$.
- C is an application cost.

Model

- A random finite economy of size I is denoted by $F^{(I)} = \{G^{(I)}, q^{(I)}, C\}$.
- $G^{(I)}$ is the random empirical distribution of types.
- $q^{(I)} = [q \cdot I]/I$ is the supply of seats per student, $f(x) = [x]$ is the floor function.
- $\hat{F}^{(I)} = \left\{\hat{G}^{(I)}, q^{(I)}, C\right\}$ is a realization of $F^{(I)}$.

MATCHING PROCESS

- Schools announce their capacities.
- Every student submits a rank-order list (ROL) of $1 \leq K_i \leq S$ schools. $L_i = \left(l_i^1, \ldots, l_i^k, \ldots, l_i^{K_i}\right)$.
- Student's true ordinal preference: $r(u_i) = \left(r_i^1, \ldots, r_i^S\right) \in \mathcal{L}$.
- Cost $C(|L|)$ depends on the number of schools being ranked in L.

Gale-Shapley deferred acceptance

Student proposing DA

Round 1 Every student applies to her first choice. Each school rejects the lowest-ranked students in excess of its capacity and temporarily holds the other students.

Round k Every student who is rejected in Round $(k-1)$ applies to the next choice on her list. Each school, pooling together new applicants and those it holds from Round $(k-1)$, rejects the lowest-ranked students in excess of its capacity. Those who are not rejected are temporarily held by the schools.

- The process terminates after any Round k when no rejections are issued.

INFORMATION STRUCTURE AND DECISION-MAKING

- Conditional on others' ROLs and priority indexes, each student i's **admission outcome** is given by

$$a_s(L_i, e_i; L_{-i}, e_{-i}) \equiv \begin{cases} \mathbf{1}\left(i \text{ is rejected by } l_i^1, \ldots, l_i^k \text{ and accepted by } l_i^{k+1} = s \mid L_i, e_i; L_{-i}, e_{-i}\right) & \text{if } s \in L_i \\ 0 & \text{if } s \notin L_i \end{cases}$$

- Given a pure strategy σ, the admission probability is the expectation of admission outcomes $\int a_s(\sigma(\theta_i), e_i; \sigma_{-i}(\theta_{-i}), e_{-i})\, dG(\theta_{-i})$
- Symmetric **equilibrium** σ^* is given by max expected utility,

$$\sigma^*(\theta_i) \in \underset{\sigma(\theta_i)\in\mathcal{L}}{\arg\max} \left\{ \sum_{s\in\mathcal{S}} u_{i,s} \int a_s\left(\sigma(\theta_i), e_i; \sigma^*_{-i}(\theta_{-i}), e_{-i}\right) dG(\theta_{-i}) - C(|\sigma(\theta_i)|) \right\}.$$

INFORMATION STRUCTURE AND DECISION-MAKING

- Cutoff of school s is given by the priority index (score) of the last student admitted or zero (if capacity is not fully occupied):

$$P_s\left(\mu\left(F^{(I)},\sigma\right)\right)=\begin{cases}\min\left\{e_{i,s}\mid\mu_{\left(F^{(l)},\sigma\right)}\left(\theta_i\right)=s\right\} & \text{if } \left|\mu^{-1}_{\left(F^{(l)},\sigma\right)}(s)\right|=q_s^{(I)}\\ 0 & \text{if } \left|\mu^{-1}_{\left(F^{(l)},\sigma\right)}(s)\right|<q_s^{(I)}\end{cases}$$

Truth telling behavior in equilibrium

- Weakly truth-telling (WTT):

$$\sigma\left(\theta_i\right) = \left(r_i^1, r_i^2, \ldots, r_i^{K_i}\right) \text{ for } K_i \leq S.$$

  This means student i ranks $K_i < S$ most-preferred schools in true preference order, but may not rank all schools.

- Strictly truth-telling (STT)

$$\sigma\left(\theta_i\right) = r\left(u_i\right)$$

  Always rank true preference order.

Truth telling behavior in equilibrium

A Problem

- DA is strategy-proof when there's no application cost, which means STT is a weakly dominant strategy for all students.
- However, it leaves the possibility of multiple equilibria. Even if all other students play STT, they have multiple best responses for student *i*.

$\Rightarrow$ We need to clarify the conditions under which STT is the unique equilibrium.

Truth telling behavior in equilibrium

An example

There are multiple equilibria in the following example.

Example 1

- Two students (i_1, i_2), three one-seat schools (s_1, s_2, s_3).
- All schools rank i_1 above i_2.
- Student i_1 's preference order is (s_1, s_2, s_3), but i_2 's is (s_2, s_1, s_3).
- For i_1, the bottom part of her submitted ROL is irrelevant as long as s_1 is top-ranked.
- For student i_2, "skipping the impossible" comes into play.

Truth telling behavior in equilibrium

Proposition 1

(Sufficiency) STT is the unique Bayesian Nash equilibrium under DA if

1. No application cost.
   - No reason to omit any schools, as marginal cost of application is 0.
2. The joint distribution of preferences and priorities G has full support.
   - In other words, this implies that the probability of being admitted by any one school is not 0, even the top school.

(Necessity) For any nonzero application cost, there always exist student types for whom STT is not an equilibrium strategy.

- As the marginal benefit of applying additional school approaches to 0.

Remark 1. Proposition 1 does not extend to WTT.

Truth telling behavior in equilibrium

Proposition 2

Under DA with application cost, if students do not play weakly dominated strategies, a student's submitted ROL is a partial order of her true preferences.

Stability

Proposition 3

Suppose that every student is WTT under DA, which may not be an equilibrium. Given a realized matching,

1. whenever a student is assigned, she is matched with her favorite feasible school;
2. if everyone who has at least one feasible school is assigned, the matching is stable.

Asymptotic Stability in Bayesian Nash Equilibrium

Assumptions:

1. Every Bayesian Nash equilibrium of the continuum economy E results in unique stable matching, μ^∞
2. There exists σ^∞ such that $\lim_{I\to\infty} G\left(\left\{\theta_i \in \Theta \mid \sigma^{(I)}(\theta_i) = \sigma^\infty(\theta_i)\right\}\right) = 1.$

ASYMPTOTIC STABILITY

DEFINITION 1

A sequence of random matchings, $\{\mu^{(I)}\}_{I\in\mathbb{N}}$, associated with the sequence of random economies and equilibrium strategies, $\{F^{(I)}, \sigma^{(I)}\}_{I\in\mathbb{N}}$, is asymptotically stable if the fraction of students who are matched with their favorite feasible school in a random finite economy $(F^{(I)})$ converges to 1 , almost surely, or, equivalently,

$$\lim_{I\to\infty} G^{(I)}\left(\left\{\theta_i \in \Theta \mid \mu^{(I)}(\theta_i) = \underset{s\in\mathcal{S}\left(e_i, P^{(I)}\right)}{\arg\max}\, u_{i,s}\right\}\right) = 1, \text{ almost surely.}$$

Explanation: As the economy enlarge, almost all students can be matched with their feasible favorite schools, almost surely.

Asymptotic stability

Proposition 4

In the sequence of random economies and equilibrium strategies, $\{F^{(I)}, \sigma^{(I)}\}_{I \in \mathbb{N}}$, if Assumptions 1 and 2 are satisfied, then

- the random cutoffs converge to those of the stable matching in the continuum economy: $\lim_{I \to \infty} P^{(I)} = P^{\infty}$, almost surely;
- the sequence of random matchings, $\{\mu^{(I)}\}_{I \in \mathbb{N}}$, is asymptotically stable.

Goals of their estimation

Building on the theoretical results:

- Formalize the estimation of student preferences under different sets of assumptions.
- Propose a series of tests to guide the selection of the appropriate approaches.

DISCRETE CHOICE AND REVEALED PREFERENCE

Student i' utility from the academic school s:

$$u_{i,s} = V_{i,s} + \epsilon_{i,s} = V\left(Z_{i,s}, \beta\right) + \epsilon_{i,s}$$

Define $Z_i \equiv \{Z_{i,s}\}_{s=1}^{S}$, and $\epsilon_i \equiv \{\epsilon_{i,s}\}_{s=1}^{s}$. It is assumed that $\epsilon_i \perp Z_i$ and that ϵ_{is} is i.i.d. over i and s with the type-I extreme value (Gumbel) distribution.

- Estimation relies on revealed preference.
- What information is revealed depends on the imposed assumption - WTT, stabulity or undominated strategies.

RP under diff assumptions

Data

Available schools: $s_1, \ldots, s_4$
Student i's priority indices: $e_{i,s} = 0.5, \forall s$
Submitted ROL of i: (s_1, s_3)
Admission outcome of i: s_3
Cutoffs: $P = (0.8, 0.9, 0.4, 0.2)$

$\Longrightarrow$

Revealed preferences

WTT: $u_{i,s_1} > u_{i,s_3} > u_{i,s}$, for $s = s_2, s_4$
Stability: $u_{i,s_3} > u_{i,s_4}$
Undominated strategies: $u_{i,s_1} > u_{i,s_3}$

Figure 1. Revealed Preferences under Different Assumptions: An Example

Back

WTT

Assumptions

- **WTT1**: Suppose $\sigma^W(u_i, e_i) = L = (l^1, \ldots, l^{K_i})$. Here, L ranks i's top K_i preferred schools according to her true preferences: $u_{i,l^1} > \cdots > u_{i,l^{K_i}} > u_{i,s'}$ for all s' not ranked in L;
- **WTT2** The number of schools ranked by a student is exogenous: $u_i \perp \left|\sigma^W(u_i, e_i)\right|, \forall i$.

WTT

Choice probability of ROL L:

$$\begin{aligned}&\Pr\left(\sigma^{W}\left(u_i,e_i\right)=L\mid Z_i;\beta\right)\\&\quad=\Pr\left(\sigma^{W}\left(u_i,e_i\right)=L\left|Z_i;\beta;\left|\sigma^{W}\left(u_i,e_i\right)\right|=K\right.\right)\times\Pr\left(\left|\sigma^{W}\left(u_i,e_i\right)\right|=K\mid Z_i;\beta\right)\end{aligned}$$

where,

$$\begin{aligned}&\Pr\left(\sigma^{W}\left(u_i,e_i\right)=L\left|Z_i;\beta;\left|\sigma^{W}\left(u_i,e_i\right)\right|=K\right.\right)\\&\quad=\Pr\left(u_{i,l'}>\cdots>u_{i,l^K}>u_{i,s'}\forall s'\in\mathcal{S}\backslash L\left|Z_i;\beta;\left|\sigma^{W}\left(u_i,e_i\right)\right|=K\right.\right)\\&\quad=\prod_{s\in\mathcal{L}}\left(\frac{\exp\left(V_{i,s}\right)}{\sum_{s'\not\succ_L s}\exp\left(V_{i,s'}\right)}\right),\end{aligned}$$

$\Rightarrow$ We can estimate $\hat{\beta}^{TT}$ using MLE or GMM given this probability.

Stability

Under the assumption of stability, the probability that i is matched with s, or chooses s in $\mathcal{S}\left(e_i, P(\mu)\right)$, is

$$\Pr\left(s = \mu\left(u_i, e_i\right) = \underset{s \in \mathcal{S}(e_i, P(\mu))}{\arg\max}\ u_{i,s} \mid Z_i, e_i, \mathcal{S}\left(e_i, P(\mu)\right); \beta\right).$$

Exogeneity of Priority Index and Feasible Set

- **EXO1** For all $i, e_i \perp \epsilon_i \mid Z_i$: Conditional on observables Z_i, student preferences and priority indices are independent.
- **EXO2** For all i and s, $\mathbf{1}\left(e_{i,s} < P_s(\mu)\right) \perp \epsilon_i \mid Z_i$, or $\mathcal{S}\left(e_i, P(\mu)\right) \perp \epsilon_i \mid Z_i$: Conditional on observables Z_i, a student's preferences and her set of feasible schools are independent.

$\Rightarrow$ Estimate $\hat{\beta}^{ST}$.
Relative to WTT, the stability assumption uses unambiguously less information from the data. Example 1

Testing Truth-Telling against Stability

Hausman Test or Hansen test

Our estimator $\hat{\beta}_{TT}$ uses all the restrictions implied by WTT. Therefore, under the null hypothesis that students are WTT, both estimators $\hat{\beta}_{TT}$ and $\hat{\beta}_{ST}$ are consistent but only $\hat{\beta}_{TT}$ is asymptotically efficient. Under the alternative that the matching is stable but not all students are WTT, only $\hat{\beta}_{ST}$ is consistent.

$$T_H = \left(\hat{\beta}_{ST} - \hat{\beta}_{TT}\right)' \left(\hat{\mathbf{v}}_{ST} - \hat{\mathbf{v}}_{TT}\right)^{-1} \left(\hat{\beta}_{ST} - \hat{\beta}_{TT}\right),$$

Undominated Strategies and Stability

When stability fails

ROLs are students' true partial preference orders. That is, a submitted ROL, L_i, respects i 's true preference order among the schools ranked in L_i .

Example 1

Integrating stability and undomminated strateies could help construct the over-identifying restrictions.

Paris school matching

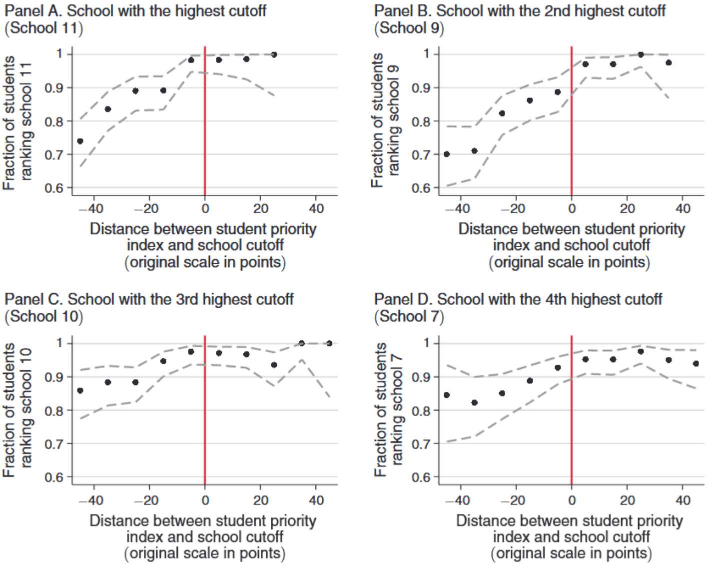

Figure 3. Fraction of Students Ranking Each of the Four Most Selective Schools in the Southern

CONCLUSION

- Truth-Telling assumption is generally violated.
- Matching outcomes are generally stable, but student ranking strategies influence the process.
- Ranking strategies are influenced by ranking constraints (ROL limits) and application costs
- Over-Identifying Restrictions (OIRs) can be used to test stability